end at $\theta^{(0)} = 0$, a perfectly good problem for a finite rod but not when there is an infinite distance between the ends.

If the problem can be approached by the asymptotic series assumed in [1], it is at best a singular perturbation where two matched expansions will be required. As noted in [1], Mahony [2] investigated the zero Grashof number limit for spheres and cylinders. He encountered the same problem we have pointed out; the far boundary condition cannot be satisfied. He overcame this difficulty by patching his solution to solutions for the far wake. His results then depend upon the method and point of patching. This curve fitting procedure is not as theoretically pleasing as a true matching process used in the method of inner and outer expansions.

It is of some interest to compute the heat transfer from the plate. The temperature gradient at the plate is

$$\frac{\partial\theta^{(0)}}{\partial y} = \frac{2/\xi_0}{[1-(2x)^2]^{\frac{1}{2}}}.$$

This is singular at the ends of the plate $x = \pm\frac{1}{2}$. However, it is integrable and the Nusselt number can be found as

$$N = \frac{hL}{k} = -\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\partial\theta^{(0)}}{\partial y} = \pi/\xi_0.$$

This result depends upon the location of the infinite boundary condition as does the temperature profile.

The artificial criteria used in [1] to compare solutions gave an answer. This result was not the proper limit as shown by the exact solution. In retrospect then, how could a researcher determine whether the real answer has been found by the computer or not? For certain problems mathematicians have produced existance and uniqueness theorems; however, more often than not, this information is missing. There appears to be no definite answer. A general idea of what to expect might be obtained by studying exact solutions for related or similar problems. In the present case a clue that the computer result is not correct is found in the solution for the cylinder given by Mahony [2].

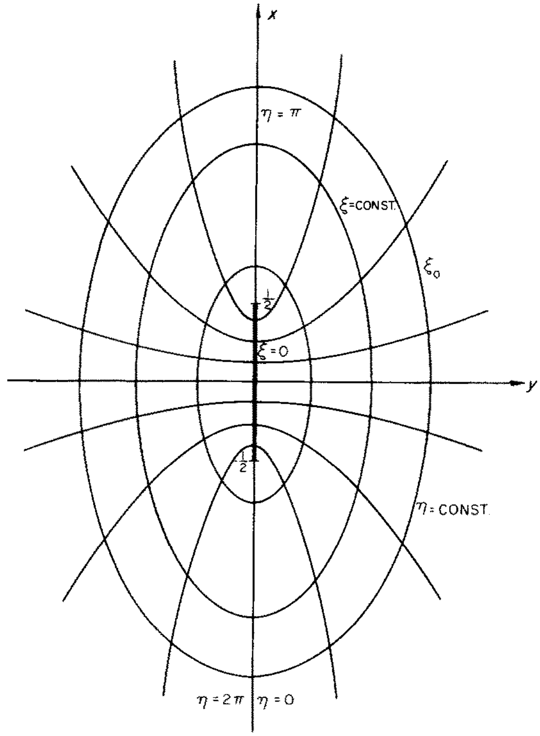

FIG. 1.

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REJOINDER

TO DISCUSS our previous paper [1], it is necessary to examine both the nature of the mathematical solution and its relevance to the real physical situation. Professor Panton's remarks on the zeroth-order or the pure conduction solution are mathematically correct and well taken. However, as we intend to show below, the difficulty pointed out by him does not affect the validity of the results of our published solution.

In view of the great complexity of the governing differential equations for laminar free convection at small Grashof numbers and the general difficulty in obtaining the solutions, theoretical studies, by necessity, must be based on certain simplifying idealizations. The problem mathematically formulated in [1] is restricted to a two-dimensional vertical finite plate in an infinite domain. It is noted that this idealization, as will be seen, has no exact counterpart in physical reality, and hence the corresponding solution, may it be exact or approximate, can be meaningful only when it reasonably approximates a real physical situation. The numerical calculations based on the relaxation technique in [1] must necessarily be confined in a finite domain, the extent of which was taken to be as large as practical and still consistent with the machine capability. This was done to minimize the effect of the field extent on the field behaviors in the immediate neighborhood of the plate, which was *the* region of interest in our study. That this is the case, may be readily demonstrated by the exact conduction solution as given by Professor Panton. The field extent used in our solution, $x = \pm 7{\cdot}5$ and $y = 7{\cdot}0$, corresponds approximately to ξ_0 of 3·33, with η taken to be $\pi/2$. This gives a Nusselt number of $N = \pi/\xi_0 = 0{\cdot}943$. When we increase the field extent by a factor of *four*, with $x = \pm 15{\cdot}0$ and $y = 14{\cdot}0$, we find that $\xi_0 = 4{\cdot}02$, which yields $N = 0{\cdot}782$. It is seen that the corresponding change in Nusselt number is only 17 per cent! Since the Nusselt number, being directly proportional to the temperature gradient, is a more critical measure of the temperature field, the above comparison signifies the fact that, with the field extent used in our study, the temperature profiles close to the plate are for practical purposes no longer sensitive to the size of the field. Thus, the effect of field extent on the results in [1] is not as severe as what one might expect after reading Professor Panton's discussion. Another measure to show the degree of adequacy of our chosen field can also be obtained from Professor Panton's solution, which gives values $\partial\theta^{(0)}/\partial y$ and $\partial^2\theta^{(0)}/\partial y^2$ at $x = 0$ and $y = 7{\cdot}0$ (or $\xi = \xi_0$ and $\eta = \pi/2$) of 0·04 and 0·006, respectively. These values tend to indicate that our choice of the locations of infinity was a reasonable one. Admittedly, the pure conduction solution obtained in [1] should only be interpreted as that for a vertical heated finite plate in a *large* but *finite* environment, instead of the infinite domain formulated originally. Even if the infinite-domain conduction problem could be solved, our solution in [1] would still be more meaningful on physical grounds, as will be discussed in the next paragraph. Once the nature of the approximation in our zeroth-order solution is understood, the higher-order perturbation quantities, as given in [1], which are driven by the zeroth-order temperature field, can be interpreted accordingly. It should be noted, however, that these perturbations are not valid at large distances away from the plate in view of the nature of the expansions.

Physically speaking, the case of laminar free convection along a vertical finite heated plate in an infinite field is not really very realistic. When the field extent is infinite, the inherent disturbances in the environment are likely to be amplified in the wake region, thus tending to lead to transition and turbulence. In such a situation, laminar flow only exists in the immediate neighborhood of the plate, and consequently the infinite domain formulation in [1] is not strictly applicable. On the other hand, it is possible to maintain steady laminar condition in the laboratory where the heated plate is suspended inside a large but finite-size container. In fact, all the known correlations for free convection to surfaces including vertical plates in the range of extremely small Rayleigh numbers (down to 10^{-7}) have been obtained in this way. For such a situation, it is not difficult to see that the solution given in [1] is actually *more* applicable than the one for the infinite-domain problem, even if such a solution should exist. However, it should also be pointed out that care must be exercised in making such a comparison in this extremely small Grashof-number range, in view of the fact that the size of the container may become a parameter.

In conclusion, we feel that on one hand we do agree with Professor Panton's discussion based on mathematical rigor. On the other hand, however, such mathematical rigor in difficult problems should be relaxed to permit obtaining meaningful informations in relevant physical situations. The analysis in [1] is a good example in which useful results are obtainable.

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HEAT TRANSFER AT THE INTERFACE OF DISSIMILAR METALS—THE INFLUENCE OF THERMAL STRAIN

In his paper, Clausing [1] explains the phenomenon of thermal rectification between dissimilar metals on the basis of thermal distortion at the interface. In order to establish theoretical values for this effect, it is necessary to solve the relevant thermoelasticity problem for non-uniform heating at a surface. A particular solution has been obtained for the